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Some Qualitative Properties of Bivariate Euler–Frobenius Polynomials

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Let M_n denote the bivariate box-spline corresponding to the directions (1, 0), (0, 1), (1, 1), each occuring with multiplicity n. We determine the critical points of the polynomials $P_n(x) = \sum_{j \in Z^2} M_n(j)\, e^{ijx}$, $n \in \mathbb{Z}_+$.

In a series of beautiful papers, Schoenberg developed the theory of univariate cardinal splines [6–8]. A basic result is the positivity of the Euler–Frobenius polynomials which implies the wellposedness of cardinal interpolation.

THEOREM 1 [6]. *Let M_r denote the univariate cardinal B-spline with support centered at* 0. *The Euler–Frobenius polynomials*

$$P_r(x) = \sum_{j \in Z} M_r(j)\, e^{ijx}, \qquad r \in \mathbb{Z}_+,$$

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are strictly positive and attain their unique minimum (maximum) at $x=\pi \pmod{2\pi\mathbb{Z}}$ $(x=0 \pmod{2\pi\mathbb{Z}})$.

In this note we obtain the bivariate analog of this result for box-splines. For a set of vectors $\Xi=\{\xi_1,\dots,\xi_n\}$ with $\xi_\nu\in\mathbb{Z}^m$, the box-spline M_Ξ is the functional on $C_0(\mathbb{R}^m)$ defined by [1]

$$M_\Xi\phi := \int_{[-1/2,\,1/2]^n} \phi\left(\sum_{\nu=1}^{n} \lambda_\nu \xi_\nu\right) d\lambda. \tag{1}$$

Equivalently, M_Ξ can be defined by its Fourier transform

$$\hat{M}_\Xi(y) = \prod_{\nu=1}^{n} S(\xi_\nu y) \tag{2}$$

where $S(z) := (2/z)\sin(z/2)$. The latter definition stresses the similarity to the univariate case. We define the multivariate Euler–Frobenius polynomials by

$$P_\Xi(x) := \sum_{j\in\mathbb{Z}^m} M_\Xi(j)\, e^{ijx}. \tag{3}$$

In the bivariate case $(m=2)$ we proved [3] the following conjecture: *The polynomials P_Ξ are strictly positive iff the box-splines $M_\Xi(\cdot-j)$, $j\in\mathbb{Z}^m$, are linearly independent.*

If valid in general $(m>2)$ the conjecture would imply that cardinal interpolation is well posed if the obvious necessary condition of linear independence is satisfied. For two variables it was shown in [2] that the box-splines are linearly independent only on the "standard" three-direction mesh, up to symmetry the vectors in Ξ have to be chosen from the set $\{(1,0), (0,1), (1,1)\}$. While the corresponding grid is very regular, the analysis of the interpolation problem is complicated. Our results [3, 4] are not as complete as in Schoenberg's univariate theory. E. g. we were not able to determine the location of the minimum for P_Ξ which in general depends on Ξ. We conjectured that in the symmetric case, when each of the three vectors in Ξ occurs with multiplicity n, the polynomial $P_n = P_\Xi$ attains its minimum at the point $(2\pi/3, 2\pi/3)$. In this note we prove this conjecture and determine all critical points of P_n.

THEOREM 2. *The polynomials P_n, $n\in\mathbb{Z}_+$, attain their minima at* $\pm(2\pi/3, 2\pi/3) \bmod 2\pi\mathbb{Z}^2$, *their maxima at the points $2\pi\mathbb{Z}^2$ and have saddle points at* $\pi\mathbb{Z}^2 \bmod 2\pi\mathbb{Z}^2$. *These are the only critical points of P_n.*

Figure 1 below shows the level curves of P_2 on $[\pi/2, 3\pi/2]\times[-\pi,\pi]$ which illustrates the general situation.

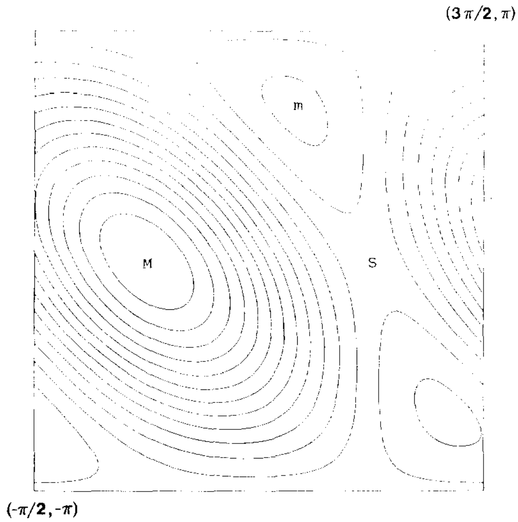

FIGURE 1

The *proof of Theorem* 2 relies heavily on the symmetries of P_n. Let $\mathscr{A}$ denote the group of 12 linear transformations which leave the mesh generated by the three directions (1, 0), (0, 1), (1, 1) invariant. This group is generated by the matrices

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \tag{4}$$

which correspond to reflection at the origin and permutation of the directions. The symmetric box-spline M_n is invariant under composition with $\mathscr{A}$, i.e.,

$$M_n(Ax) = M_n(x), \qquad A \in \mathscr{A}. \tag{5}$$

Therefore, the corresponding Euler–Frobenius polynomials satisfy

$$P_n(A^*x + 2\pi j) = P_n(x), \qquad A \in \mathscr{A}, j \in \mathbb{Z}^2, \tag{6}$$

where A^* denotes the transpose of A. These relations give much information about the structure of P_n. Denote by $\nabla f(u, v) := (D_u f(u, v), D_v f(u, v))$ the gradient of a function f. Differentiating identity (6) we obtain

$$(\nabla P_n(A^*x + 2\pi j))\, A^* = \nabla P_n(x), \qquad A \in \mathscr{A}, j \in \mathbb{Z}^2. \tag{7}$$

Let I denote the unit matrix. Identity (7) implies in particular that

$$\nabla P_n(x) \in \ker(I - A) \qquad \text{if} \quad (I - A^*)\, x = 2\pi j. \tag{8}$$

For $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} \in \mathscr{A}$ it follows from (8) that ∇P_n vanishes at the points $\pi\mathbb{Z}^2$ and $\pm(2\pi/3, 2\pi/3) + 2\pi\mathbb{Z}^2$, respectively. For $A = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, $\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \in \mathscr{A}$, the matrices $(I - A)$ have rank one and it follows from (8) that for $k \in \mathbb{Z}$,

$$\begin{aligned}
(0, 1)\,\nabla P_n(x) &= 0 \qquad \text{if} \qquad (1, 2)\, x = 2\pi k,\\
(1, 0)\,\nabla P_n(x) &= 0 \qquad \text{if} \qquad (2, 1)\, x = 2\pi k,\\
(1, -1)\,\nabla P_n(x) &= 0 \qquad \text{if} \quad (1, -1)\, x = 2\pi k,\\
(1, 1)\,\nabla P_n(x) &= 0 \qquad \text{if} \qquad (1, 1)\, x = 2\pi k,\\
(2, -1)\,\nabla P_n(x) &= 0 \qquad \text{if} \qquad (1, 0)\, x = 2\pi k,\\
(1, -2)\,\nabla P_n(x) &= 0 \qquad \text{if} \qquad (0, 1)\, x = 2\pi k.
\end{aligned} \tag{9}$$

The remaining four matrices in $\mathscr{A}$ give no further information.

Let Ω denote the (closed) triangle with vertices $(0, 0)$, $(\pi, 0)$, $(2\pi/3, 2\pi/3)$. The set

$$\Omega^* := \bigcup_{A \in \mathscr{A}} A\Omega,$$

which is the convex hull of the six points $\pm(2\pi/3, 2\pi/3)$, $\pm(4\pi/3, -2\pi/3)$, $\pm(2\pi/3, -4\pi/3)$, is a fundamental domain, i.e., its translates form an essentially disjoint partition of $\mathbb{R}^2$. Therefore, to complete the proof of Theorem 2, it is sufficient to show that

$$\nabla P_n(x) \neq 0 \qquad \text{for} \quad x \in \Omega \backslash \{(0, 0), (\pi, 0), (2\pi/3, 2\pi/3)\} \tag{10}$$

and that

$$P_n(2\pi/3, 2\pi/3) < P_n(\pi, 0) < P_n(0, 0). \tag{11}$$

To this end we proof the following estimates (Fig. 2):

(i) $D_u P_n(u, v)/(2u + v) < 0$ for $(u, v) \in \Omega_1 := \{(u, v)\colon \ 0 \leqslant v \leqslant u, 2u + v \leqslant 3\pi/2, \ u > 0\}$,

(ii) $D_u P_n(u, v)/(2\pi - 2u - v) < 0$ for $(u, v) \in \Omega_2 := \{(u, v)\colon \ 3\pi/2 \leqslant 2u + v < 2\pi, \ 0 \leqslant v, \ u + 2v \leqslant 2\pi\}$,

(iii) $D_v P_n(u, v)/v < 0$ for $(u, v) \in \Omega_3 := \{(u, v)\colon 0 < v = 2\pi - 2u \leqslant \pi/3\}$.

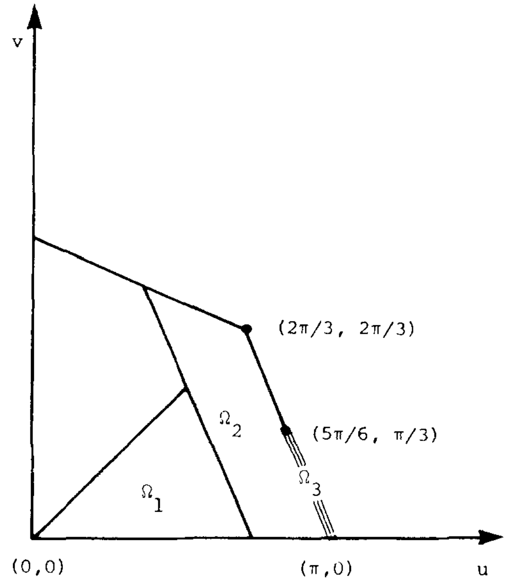

FIGURE 2

Note that, since $P_n(u, v) = P_n(v, u)$, it follows from (ii) that

$$D_v P_n(\pi - v/2, v)/(\pi - 3v/2) < 0, \qquad \pi/3 \leqslant v < 2\pi/3.$$

For small n the inequalities (i)–(iii) can be verified numerically and we shall assume in the sequel that n is sufficiently large ($n \geqslant 5$). Using the Poisson summation formula and (2), we write P_n in the form

$$P_n(u, v) = \sum_{(k, l) \in \Lambda} S(u+k)^n \, S(v+l)^n \, S(u+v+k+l)^n \tag{12}$$

where $\Lambda := 2\pi\mathbb{Z}^2$. For $(u, v) \in \Omega$ and large n, the terms with $|k| + |l|$ small dominate in the expression for P_n. This fact is crucial for the subsequent estimates.

Proof of (i). We write

$$D_u P_n(u, v) = n \sum_{(k, l) \in \Lambda} a_{k,l} b_{k,l} \tag{13}$$

with

$$\begin{aligned} a_{k,l} &:= S(u+k)^{n-1} \, S(v+l)^n \, S(u+v+k+l)^{n-1} \\ b_{k,l} &:= S'(u+k) \, S(u+v+k+l) + S(u+k) \, S'(u+v+k+l). \end{aligned} \tag{14}$$

Using the inequalities

$$|S(w)|,\ |S'(w)| \leqslant \min(1, 2/|w|), \tag{15}$$
$$-\tfrac{1}{12}w \leqslant S'(w) \leqslant -\tfrac{1}{16}w, \qquad 0 \leqslant w \leqslant \pi,$$

for $(u, v) \in \Omega_1$, we obtain the estimates

$$\begin{aligned} b_{0,0} &\geqslant -\frac{1}{12}(2u+v), \\ b_{0,0} &\leqslant -\frac{1}{16}uS(\pi) - \frac{1}{16}(u+v)\,S\left(\frac{3\pi}{4}\right) \leqslant -\frac{1}{8\pi}(2u+v), \\ \left|\frac{b_{k,l}}{b_{0,0}}\right| &\leqslant \frac{4\cdot 8\pi}{|u+k|\,|u+v+k+l|}\left(\frac{\sin(u/2)}{2u+v} + \frac{\sin((u+v)/2)}{2u+v}\right) \\ &\leqslant \frac{16\pi}{|u+k|\,|u+v+k+l|}. \end{aligned} \tag{16}$$

For $(u, v) \in \Omega_1$ and $(k, l) \neq (0, 0)$, we have

$$\frac{1}{|u+k|\,|u+v+k+l|}\left|\frac{v}{v+l}\right| \leqslant \pi^{-2}.$$

Combining this inequality with (16), we see from the definition of $a_{k,l}$ and S that

$$\begin{aligned} \left|\frac{D_u P_n(u, v)}{n a_{0,0} b_{0,0}} - 1\right| &\leqslant \sum_{A\setminus(0,0)} \left|\frac{a_{k,l}}{a_{0,0}}\right| \left|\frac{b_{k,l}}{b_{0,0}}\right| \\ &\leqslant \sum \left|\frac{u}{u+k}\right|^{n-1} \left|\frac{v}{v+l}\right|^{n} \left|\frac{u+v}{u+v+k+l}\right|^{n-1} \frac{16\pi}{|u+k|\,|u+v+k+l|} \\ &\leqslant \frac{16}{\pi}\sum \left|\frac{3\pi/4}{3\pi/4+k}\right|^{n-1} \left|\frac{\pi/2}{\pi/2+l}\right|^{n-1} \left|\frac{\pi}{\pi+k+l}\right|^{n-1}. \end{aligned}$$

The last right-hand side is less than 1 for $n \geqslant 5$. Therefore, inequality (i) follows from the second inequality in (16) and the fact that $a_{0,0}$ is positive on Ω_1.

Proof of (ii). In expression (13) for $D_u P_n$ we split the index set A into the three parts:

$$\begin{aligned} A_0 &:= \{(k, l)\colon 2k + l + 2\pi = 0\}, \\ A_\pm &:= \{(k, l)\colon \pm(2k + l + 2\pi) > 0\}. \end{aligned}$$

The sets Λ_+ and Λ_- are related by the bijective mapping

$$(k, l) \in \Lambda_+ \leftrightarrow (k', l') = (-k-l-2\pi, l) \in \Lambda_- .$$

Therefore, we can write $D_u P_n$ in the form

$$D_u P_n(u, v) = n \sum_{\Lambda_0} a_{k,l} b_{k,l} + n \sum_{\Lambda_+} a_{k,l} \tilde{b}_{k,l} \tag{17}$$

where (cf. (14))

$$\begin{aligned} \tilde{b}_{k,l} &:= b_{k,l} + \frac{a_{-k-l-2\pi,l}}{a_{k,l}} b_{-k-l-2\pi,l} \\ &= [S'(u+k)\, S(u+v+k+l) + S(u+k)\, S'(u+v+k+l)] \\ &\quad + \zeta^{n-1}[S'(u-k-l-2\pi)\, S(u+v-k-2\pi) \\ &\qquad + S(u-k-l-2\pi)\, S'(u+v-k-2\pi)] \end{aligned}$$

with

$$\zeta := \frac{u+k}{u-k-l-2\pi} \frac{u+v+k+l}{u+v-k-2\pi}.$$

Observe that for $(u, v) \in \Omega_2$ and $(k, l) \in \Lambda_+$,

$$0 \leqslant \zeta \qquad \text{and} \qquad 1 - \zeta = \frac{2\pi - v - 2u}{u-k-l-2\pi} \frac{2k+l+2\pi}{u+v-k-2\pi}. \tag{18}$$

Since the numerator in $1-\zeta$ is positive, letting $\Lambda_* := \{(k, l) \in \Lambda_+ : (k+l+\pi)(k+\pi) > 0\}$, we have

$$\begin{aligned} &0 \leqslant \zeta \leqslant 1, && (k, l) \in \Lambda_*, \\ &0 \leqslant 1/\zeta \leqslant 1, && (k, l) \in \Lambda_+ \setminus \Lambda_*. \end{aligned} \tag{19}$$

Using the identity

$$S(p)\, S'(q) \pm S'(p)\, S(q) = \frac{2}{pq} \sin \frac{p \pm q}{2} - \frac{4(p \pm q)}{p^2 q^2} \sin \frac{p}{2} \sin \frac{q}{2}, \tag{20}$$

we can simplify the above expressions for $b_{k,l}$ and $\tilde{b}_{k,l}$ and obtain

$$\begin{aligned} b_{k,l} = &\frac{2 \sin(u + v/2 - \pi)}{(u+k)(u+v+k+l)} \\ &- \frac{4(2u+v-2\pi)}{(u+k)^2\,(u+v+k+l)^2} \sin \frac{u+k}{2} \sin \frac{u+v+k+l}{2}, \qquad (k, l) \in \Lambda_0, \end{aligned} \tag{21}$$

$$\tilde{b}_{k,l} = \frac{2(-1)^l \sin(u+v/2)}{(u+k)(u+v+k+l)}(1+\zeta^n)$$
$$-\frac{4(-1)^l \sin(u/2)\sin((u+v)/2)}{(u+k)^2\,(u+v+k+l)^2}$$
$$\times\,[(2u+v+2k+l)+\zeta^{n+1}(2u+v-2k-l-4\pi)], \qquad (k,l)\in\Lambda_*. \tag{22}$$

In the term in square brackets we add and subtract $(2u+v-2k-l-4\pi)$. Then a direct computation using (18) yields

$$[\cdots] = (2u+v-2\pi)\left(2+\frac{(2k+l+2\pi)(2u+v-2k-l-4\pi)}{(u-k-l-2\pi)(u+v-k-2\pi)}\sum_{\nu=0}^{n}\zeta^{\nu}\right). \tag{23}$$

Analogous to case (i) we show that $a_{0,0}\tilde{b}_{0,0}$ is the dominant term for the right-hand side of (17). Indeed,

$$\tilde{b}_{0,0} \leqslant -0.6\,\frac{2\pi-2u-v}{u(u+v)} \qquad \text{for} \quad n\geqslant 5, \tag{24}$$

as one checks numerically for $n=5$, and therefore has it for $n\geqslant 5$, since $\tilde{b}_{0,0}$ decreases as n increases as we see from (19), (22), and (23). For $(u,v)\in\Omega_2$ we have $\pi/3\leqslant u$, $u+v\leqslant 4\pi/3$ and we obtain from (19)–(24) the estimates

$$\left|\frac{b_{k,l}}{\tilde{b}_{0,0}}\right| \leqslant \frac{2}{0.6}\left|\frac{u}{u+k}\right|\left|\frac{u+v}{u+v+k+l}\right|, \qquad (k,l)\in\Lambda_0, \tag{25}$$

$$\left|\frac{\tilde{b}_{k,l}}{\tilde{b}_{0,0}}\right| \leqslant \frac{3(n+1)(2k+l+2\pi)}{0.6}\left|\frac{u}{u+k}\right|\left|\frac{u+v}{u+v+k+l}\right|, \qquad (k,l)\in\Lambda_*. \tag{26}$$

For $(k,l)\in\Lambda_+\backslash\Lambda_*$ we estimate $\zeta^{-n}\tilde{b}_{k,l}$ in a similar way and obtain

$$\left|\frac{\tilde{b}_{k,l}}{\tilde{b}_{0,0}}\right| \leqslant \zeta^n\,\frac{3(n+1)(2k+l+2\pi)}{0.6}\left|\frac{u}{u+k}\right|\left|\frac{u+v}{u+v+k+l}\right|, \qquad (k,l)\in\Lambda_+\backslash\Lambda_*. \tag{27}$$

For $(k,l)=(0,-2\pi)$ we obtain the sharper estimate

$$\left|\frac{b_{0,-2\pi}}{\tilde{b}_{0,0}}\right| \leqslant 0.6\,\frac{u+v}{2\pi-u-v}, \qquad n\geqslant 5, \tag{28}$$

numerically for $n=5$, hence valid for $n\geqslant 5$ since $|\tilde{b}_{0,0}|$ increases with n.

Similarly as for case (i), it follows from (17), (25)–(28), the definition of ζ, and the inequality

$$(v/(2\pi - v))^n \, ((u+v)/(2\pi - u - v))^n \leqslant 1, \qquad (u, v) \in \Omega_2,$$

that

$$\left| \frac{D_u P_n(u, v)}{n a_{0,0} \bar{b}_{0,0}} - 1 \right|$$
$$\leqslant 0.6 + \frac{2}{0.6} \sum_{\Lambda_0 \setminus (0, -2\pi)} \left| \frac{\pi}{\pi + k} \right|^n \left| \frac{5\pi/6}{5\pi/6 + l} \right|^n \left| \frac{4\pi/3}{4\pi/3 + k + l} \right|^n$$
$$+ \frac{3(n+1)}{0.6} \sum_{\Lambda_* \setminus (0,0)} \left| \frac{\pi}{\pi + k} \right|^n \left| \frac{5\pi/6}{5\pi/6 + l} \right|^n \left| \frac{4\pi/3}{4\pi/3 + k + l} \right|^n (2k + l + 2\pi)$$
$$+ \frac{3(n+1)}{0.6} \sum_{\Lambda_+ \setminus \Lambda_*} \left| \frac{\pi}{\pi + k + l} \right|^n \left| \frac{5\pi/6}{5\pi/6 + l} \right|^n \left| \frac{4\pi/3}{2\pi/3 + k} \right|^n (2k + l + 2\pi).$$

The right-hand side is less than 1 for $n \geqslant 5$ and the inequality (ii) follows from (24) and the fact that $a_{0,0}$ is positive.

Proof of (iii). We have

$$D_v P_n(\pi - v/2, v) = n \sum_{\Lambda} a'_{k,l} b'_{k,l} \tag{29}$$

with

$$a'_{k,l} := S(\pi - v/2 + k)^{n-1} \, S(v + l)^{n-1} \, S(\pi + v/2 + k + l)^{n-1}$$
$$b'_{k,l} := S(\pi - v/2 + k) \, S'(v + l) \, S(\pi + v/2 + k + l)$$
$$+ S(\pi - v/2 + k) \, S(v + l) \, S'(\pi + v/2 + k + l).$$

Note that $a_{0,0} = a_{-2\pi,0}$. It can be verified numerically that

$$C := - \sup_{0 < v \leqslant \pi/3} (b'_{0,0} + b'_{-2\pi,0})/v \geqslant 0.1. \tag{30}$$

To estimate the remaining terms in (29) we observe from the definition of S and (15) that

$$|b'_{k,l}| \leqslant 2v^2 \min \left\{ 1, \frac{2}{|\pi - v/2 + k| \; |v + l| \; |\pi + v/2 + k + l|} \right\}.$$

Therefore,

$$\left| \frac{a'_{k,l} b'_{k,l}}{a'_{0,0} C v} \right| \leqslant \frac{1}{0.2} \left| \frac{\pi}{\pi + k} \right|^n \left| \frac{\pi/3}{\pi/3 + l} \right|^n \left| \frac{7\pi/6}{7\pi/6 + k + l} \right|^n,$$

and we obtain

$$\left|\frac{D_v P_n(\pi - v/2, v)}{n a_{0,0}(b_{0,0} + b_{-2\pi,0})} - 1\right| \leqslant \frac{1}{0.2} \sum_{A \setminus \{(0,0),(-2\pi,0)\}} \left|\frac{\pi}{\pi + k}\right|^n \left|\frac{\pi/3}{\pi/3 + l}\right|^n \left|\frac{7\pi/6}{7\pi/6 + k + l}\right|^n.$$

The right-hand side is less than 1 for $n \geqslant 2$ which, together with (30), implies the inequality (iii).

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